ANDHRA UNIVERSITY
TRANS-DISCIPLINARY RESEARCH HUB

TOPOLOGY

UNIT - I :

Convergence : Sequences and Nets - Filterbases in Spaces - Convergence Properties of Filterbases - Closure in terms of Filterbases - Continuity; Convergence in Cartesian Products - Adequacy of Sequences - Maximal Filterbases (Chapter X of the Prescribed Text Book).

UNIT – II :

Compactness : Compact Spaces – Special Properties of Compact Spaces – Countable Compactness - Compactness in Metric Spaces – Perfect Maps – Local Compactness - σ - Compact Spaces – Compactification – k- Spaces – Bair Spaces; Category (Chapter XI of the Prescribed Text Book).

UNIT – III :

Function Spaces : The Compact - open Topology – Continuity of Composition; the Evaluation Map – Cartesian Products – Application to Identification Topologies – Basis for Z^Y – Compact Subsets of Z^Y – Sequential Convergence in the c -Topology – Metric Topologies; Relation to the c -Topology – Pointwise Convergence – Comparison of Topologies in Z^Y (Chapter XII of the Prescribed Text Book).

UNIT – IV :

Complete Spaces : Cauchy Sequences – Complete Metrics and Complete Spaces – Cauchy Filterbases; Total Boundedness – Baire's Theorem for Complete Metric Spaces – Extension of Uniformly Continues Maps – Completion of a Metric Space – Fixed- Point Theorem for Complete Spaces – Complete Subspaces of Complete Spaces – Complete Gauge Structures (Chapter XIV of the Prescribed Text Book).

Prescribed Text Book : James Dugundji, Universal Book Stall, New Delhi.

Reference Text Books : 1. John L Kelly, General Topology, D. Van Nostrand Company, Inc. 120 Alexandar St Princeton, New Jersey. 24 West 40th Street, New York 18, New York.
2. Bourbaki, General Topology, Addison – Wesley Publishing Company, London.

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MODEL PAPER

Time : 3 hours **Max.Marks :100**

Answer any FIVE questions, All questions carry equal marks.

1. (a)Prove that a topological space Y is Hausdorff if and only if each convergent Filterbase in Y converges to exactly one point.

(b) Let Y be a topological space and $A \subset Y$. Then prove that $y \in \bar{A}$ if and only if there is a filterbase on A converging to y.

2. (a) Let X be 1^o countable and let $A \subset X$. Then prove that $x \in \bar{A}$ if and only if there is a sequence on A converging to x.

(b) Let B be any filterbase in Y. Then prove that there exists a maximal filterbase $M \vdash B$

3. (a) Let Y be compact, Z Hausdorff, and p: Y x Z $\rightarrow$ Z , the projection "parallel to the compact factor Y."Then prove that p is a closed map.

(b) Let X beHausdorf and Y be compact. Then prove that f: X $\rightarrow$ Y is continuous if and only if its graph G(f) is closed in X x Y.

4.(a) Prove that a space is compact if and only if it is both countably compact and metacompact.

(b) Prove that a countably compact space Y is metrizable if and only if it is 2^o countable.

5. (a) Let X, Z be Hausdorff and Y locally compact. Then prove that the map T: $Y^X \times Z^Y \rightarrow Z^X$ iscontinuous.

(b) Let X be a k – space and Y a locally compact space. Then prove that X x Y is a k- space.

6. (a) Let Y be locally compact, and X, Z arbitrary. Then prove that the map $\alpha \rightarrow \hat{\alpha}$ establishes a homeomorphism of $Z^{X \times Y}$ and $(Z^y)^x$.

(b) Let (Z,d) be a metric space and Y an arbitrary space. Then prove that a sequence $\{f_n\}$ in Z^Y converges to an $f \in Z^Y$ uniformly on every compact subset if and only if $f_n \rightarrow f$ in the c – topology of Z^Y.

7. (a) Let X be an arbitrary space, and let Y be d – complete. Then prove that C(X, Y; d) is d^+ - complete.

(b) Prove that ametrizable space Y is compact if and only if it has a metric d that is both complete andtotally bounded.

8. (a) Prove that any topologically complete space is a Baire space.

(b) Let Y be d – complete, and let T: Y $\rightarrow$Ybe d-contractive. Then prove that T is continuous and has exactly one fixed point.